Universal System Simulation via Hypergraphs

John Morris

Dept. of Mechanical Engineering
Clemson University

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Clemson Engineering Design Applications and Research

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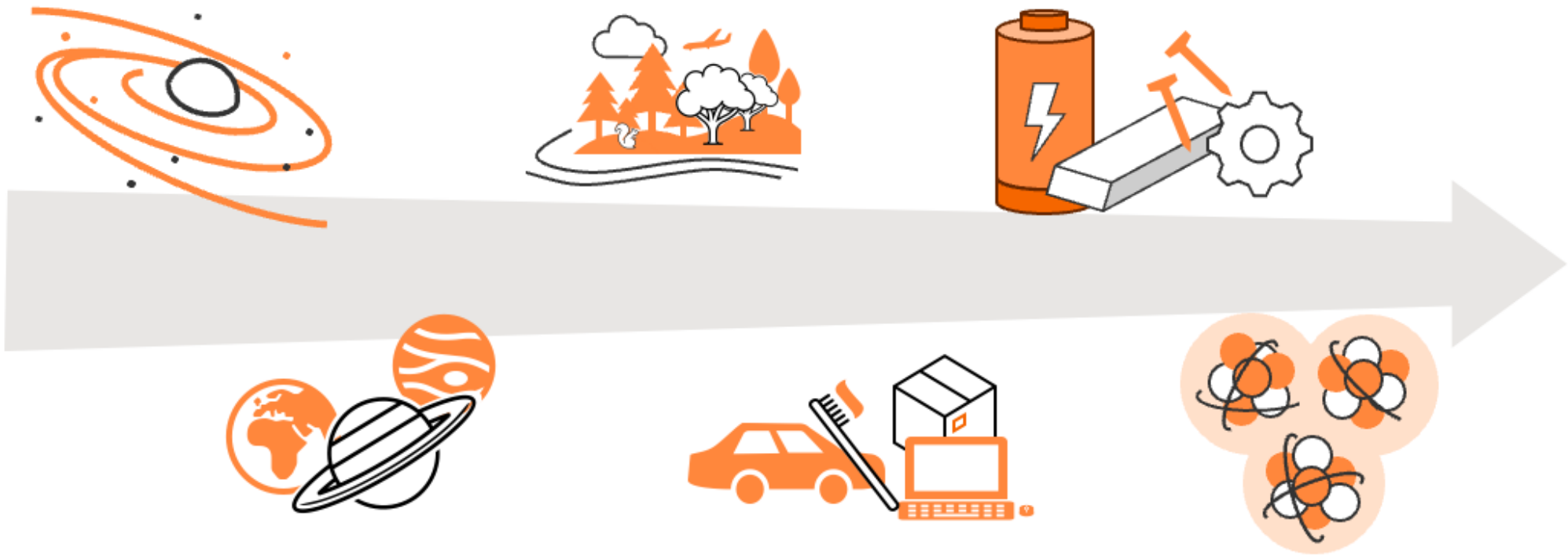
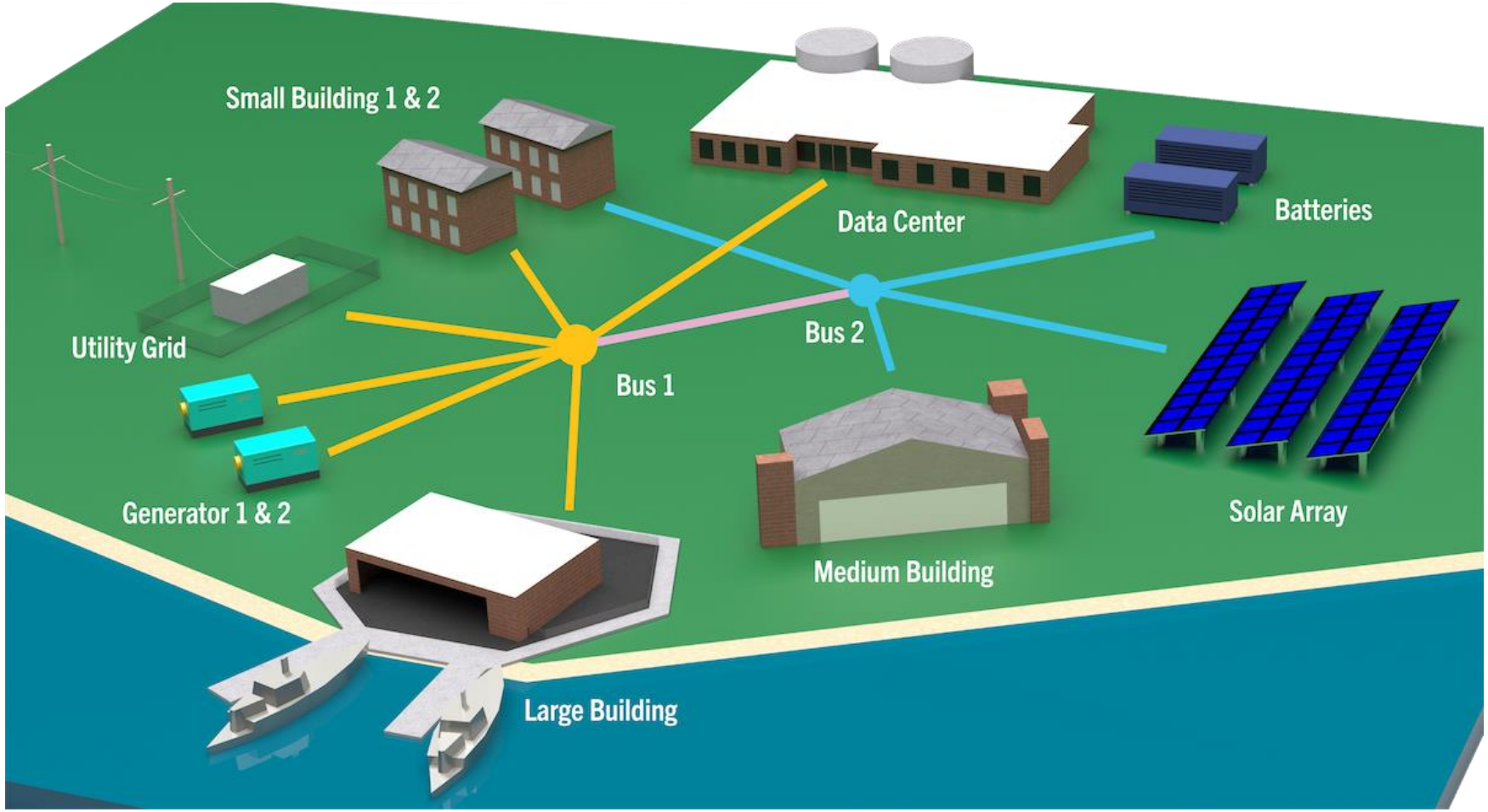
Science is the work of characterizing the systems around us

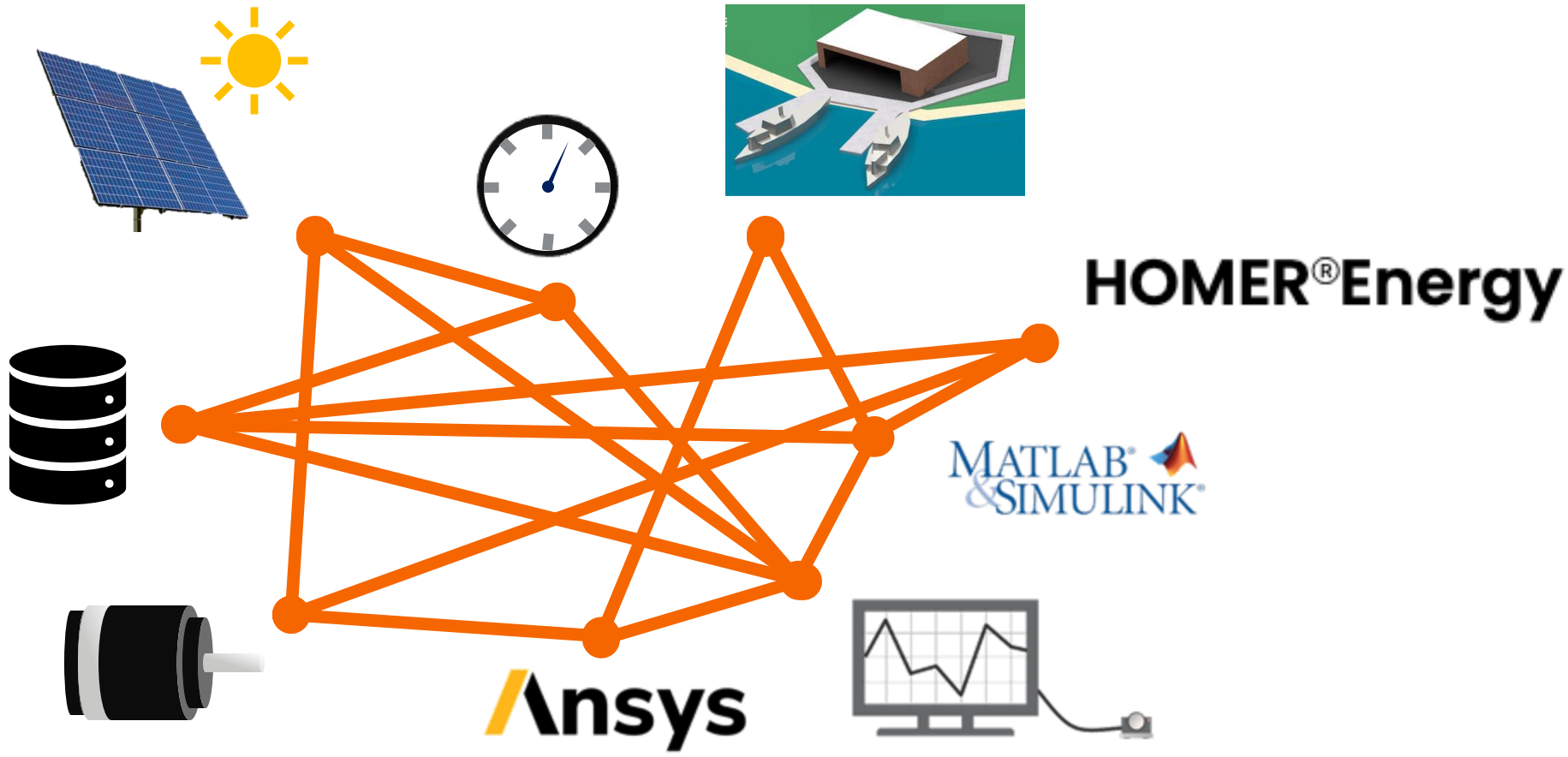


Goal of an agent is to observe facts about a system (its state)

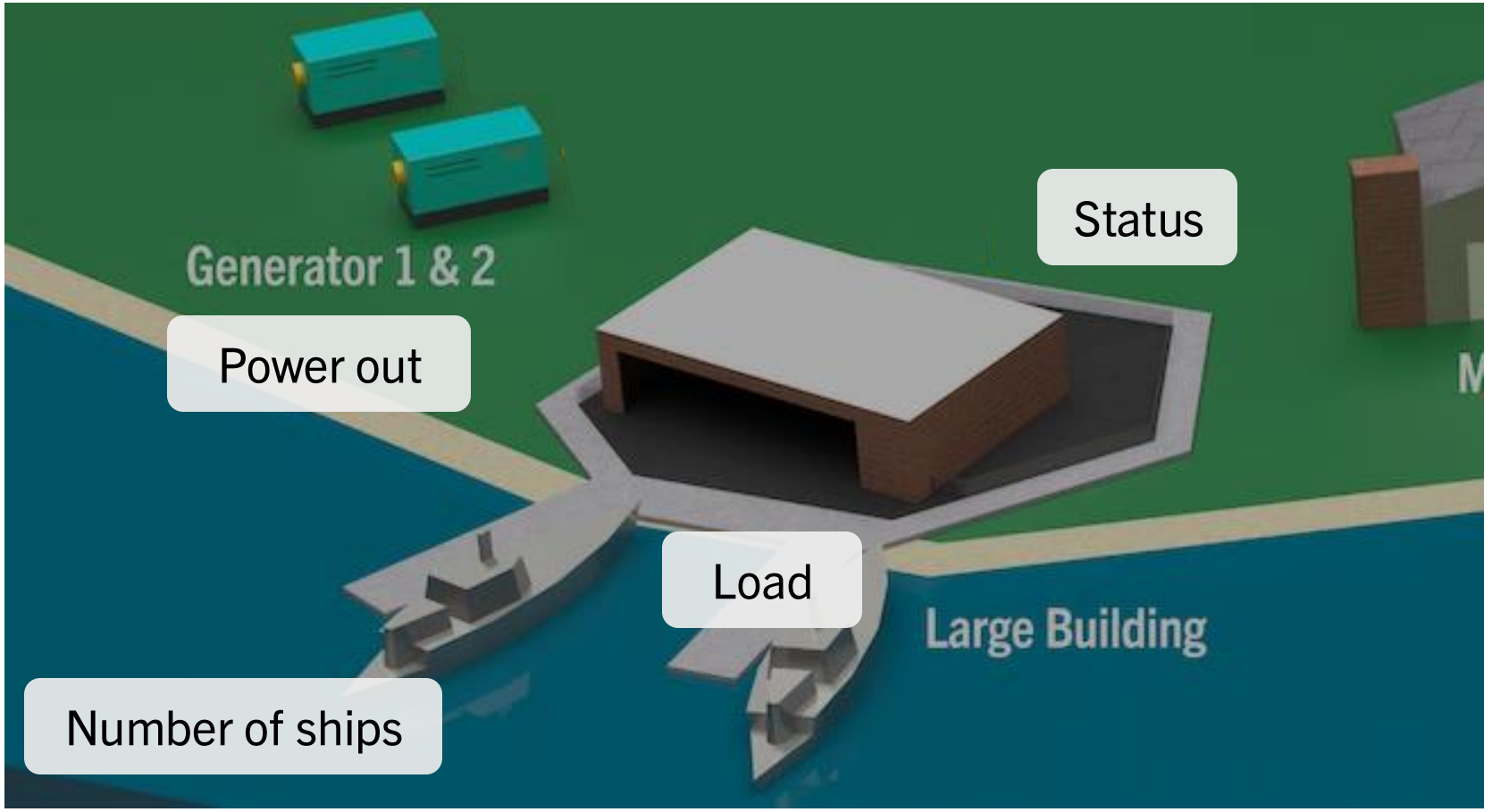
Example: Naval Microgrid



Complex systems requires models to be represented in different formalisms and software

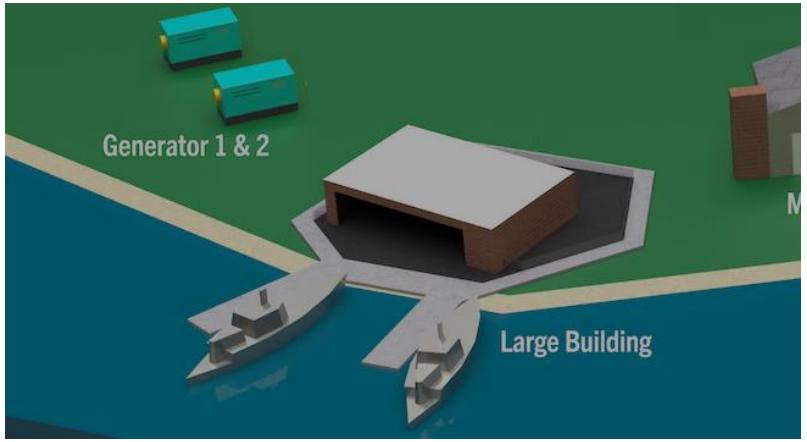


“A system is a collection of variables” (Ross Ashby)



Ashby, W. R. (1956). An Introduction to Cybernetics. Chapman & Hall.

States are sets of values representing specific properties

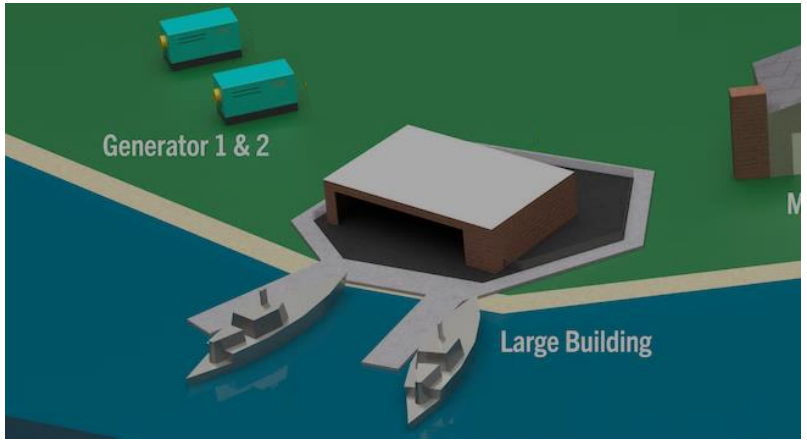


Number of ships	Load	Power out	Status

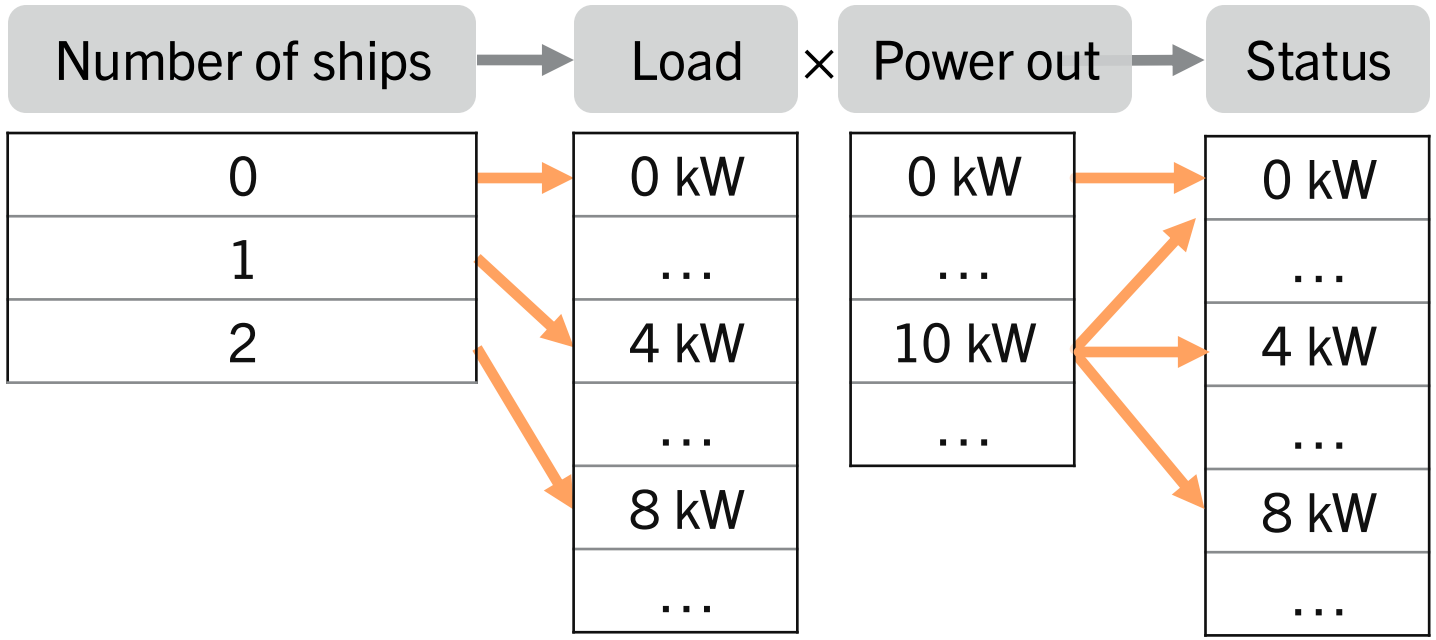
$$\mathbb{N}_2 \times \mathbb{R} \times \mathbb{R}_{\geq 0} \times \mathbb{R}$$

Polderman, J. W., & Willems, J. C. (1998). Dynamical Systems. In Introduction to Mathematical Systems Theory: A Behavioral Approach. Springer.

Behaviors are reductions of the set of possible configurations, forming a model



$f(n_{ships}) \rightarrow L \coloneqq 4{,}000 n_{ships}$ $\quad$ $f(p_o, L) \rightarrow S \coloneqq \min(p_o, L)$



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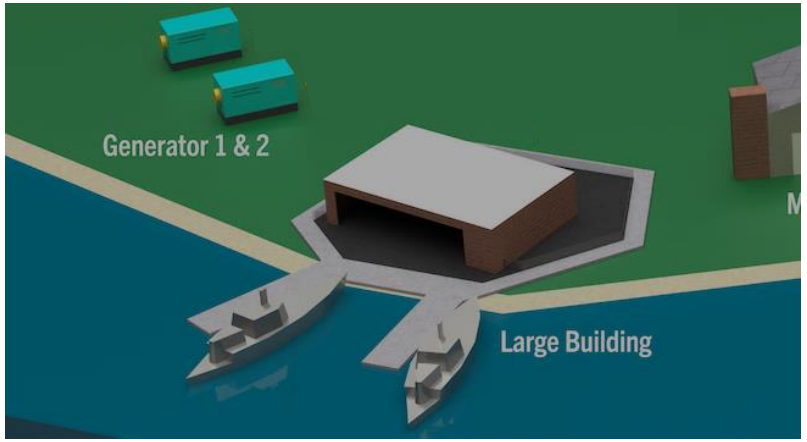
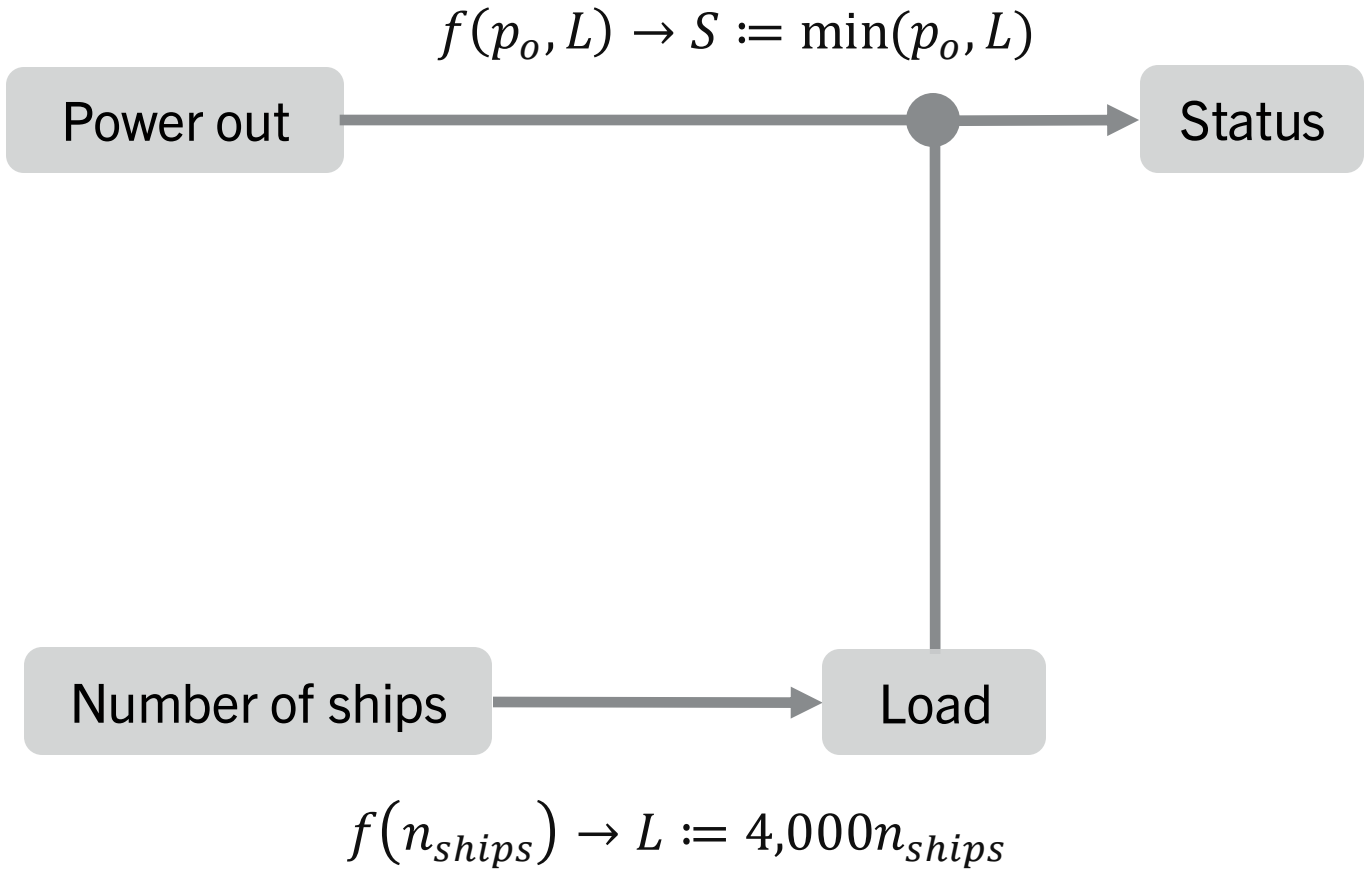
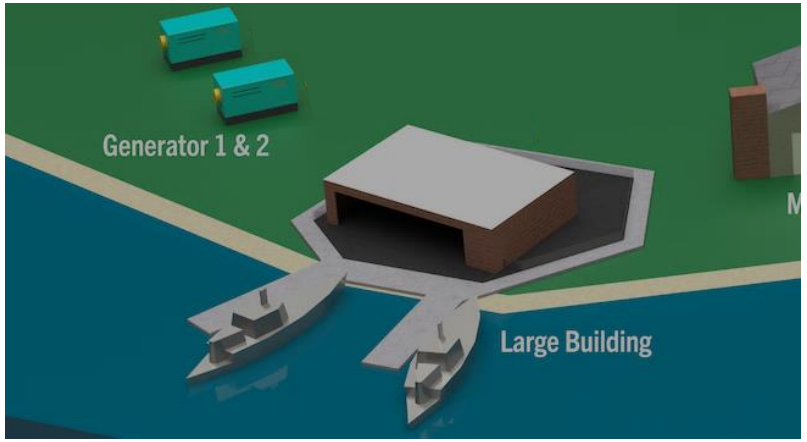
Simple (pragmatic) model of a system is a hypergraph





Schultz, P., Spivak, D. I., & Vasilakopoulou, C. (2019). Dynamical Systems and Sheaves (No. arXiv:1609.08086). arXiv. https://doi.org/10.48550/arXiv.1609.08086

There are 2 mechanisms for observing system states: coupling and simulation



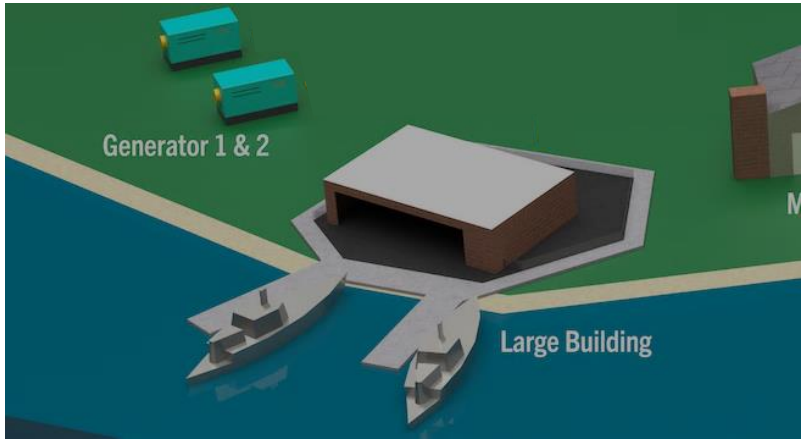
$$f(p_o, L) \rightarrow S \coloneqq \min(p_o, L)$$

Power out → Status

Number of ships → Load

$$f(n_{ships}) \rightarrow L \coloneqq 4{,}000 n_{ships}$$

Coupling: synchronizing the states of the system are with the observer



$$f(p_o, L) \rightarrow S \coloneqq \min(p_o, L)$$

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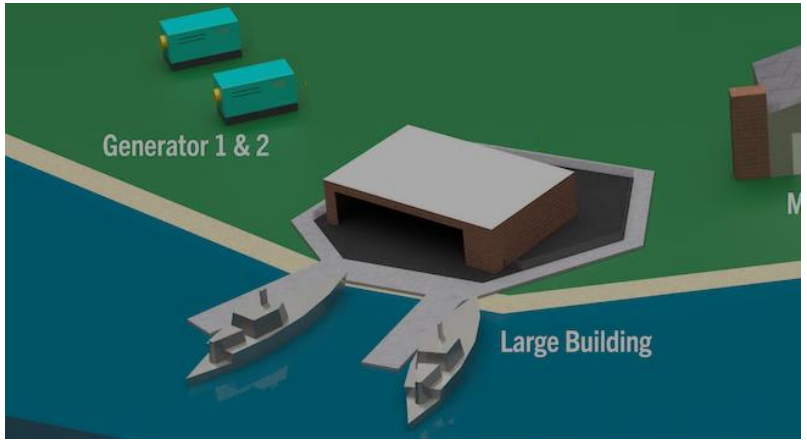
Simulation: approximation by relating inputs to outputs



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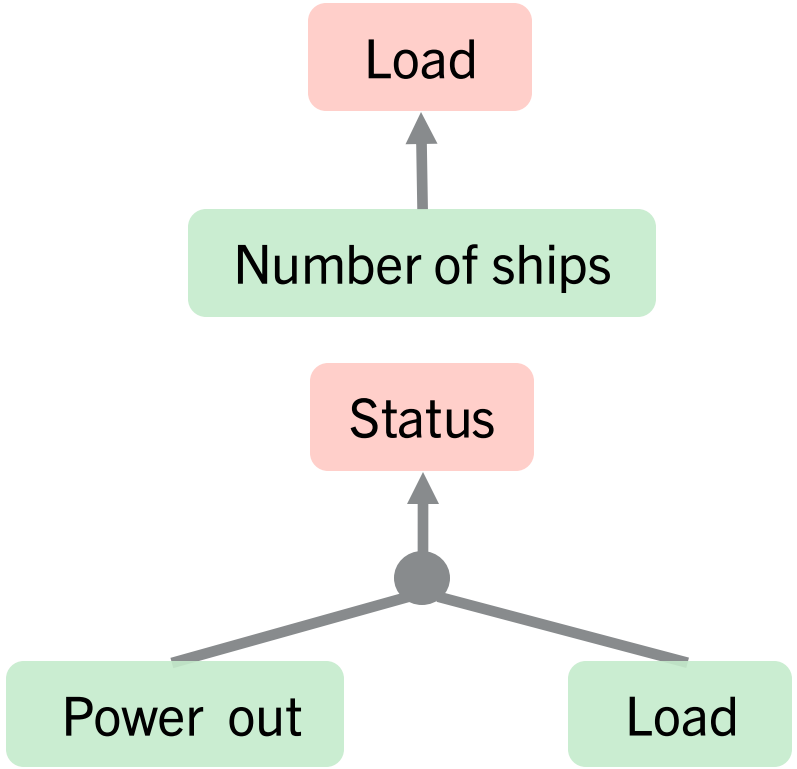
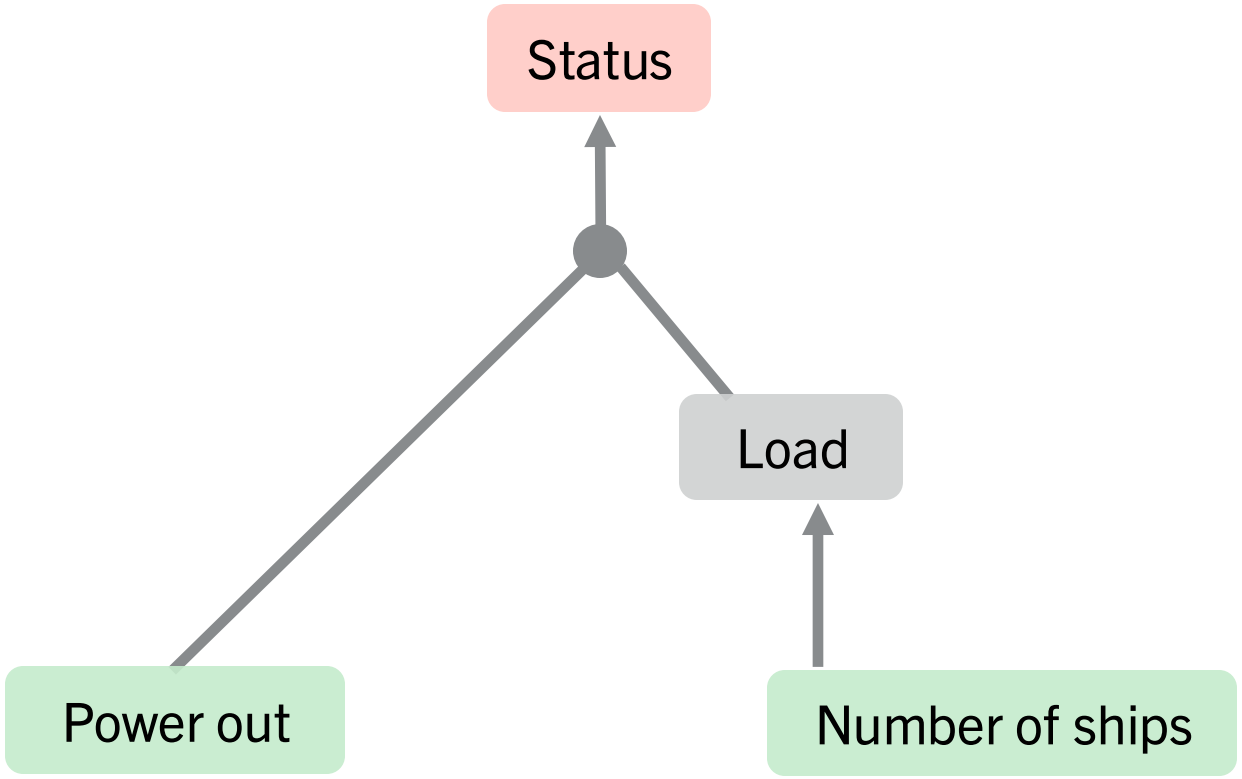
All possible simulations pairing inputs to outputs are given by the set of paths through the hypergraph

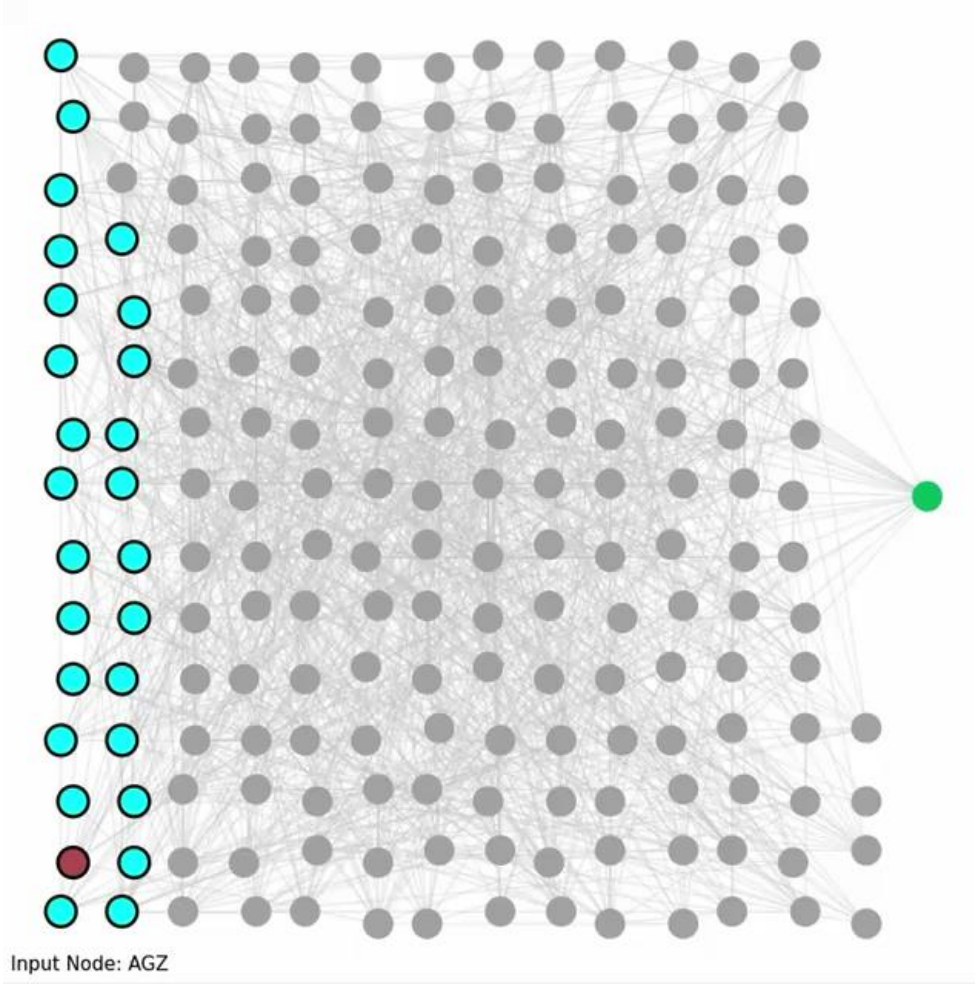




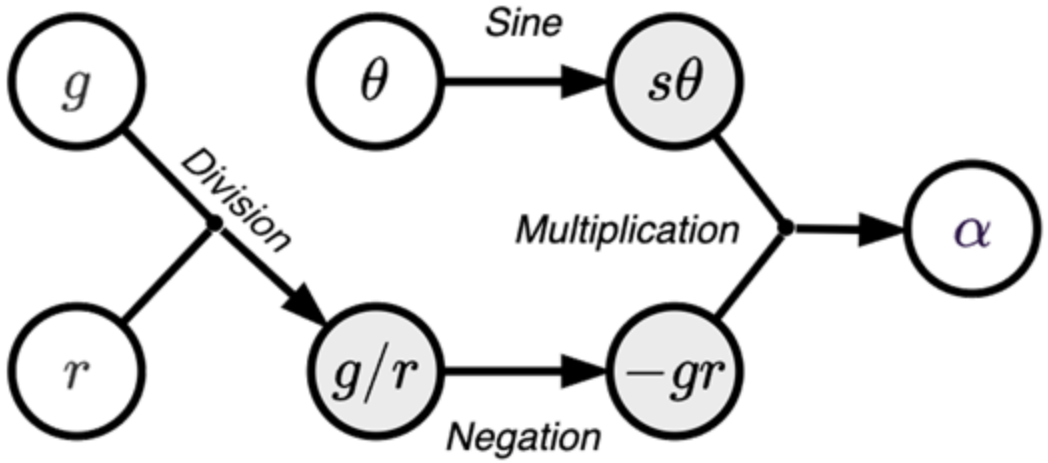
Robinson, M. (2017). Sheaf and Duality Methods for Analyzing Multi-Model Systems. In Recent Applications of Harmonic Analysis to Function Spaces, Differential Equations, and Data Science. Springer International Publishing.

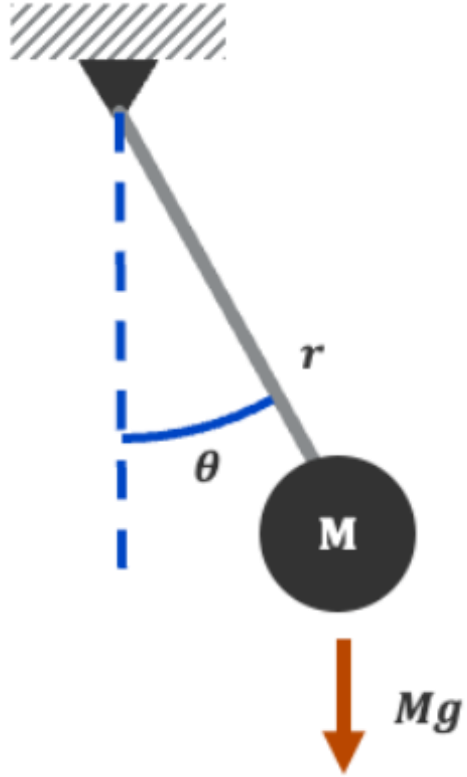
Solving engine available on the Python Package Index

Implements a Breadth-First Search to simulate a system



Software Demonstration

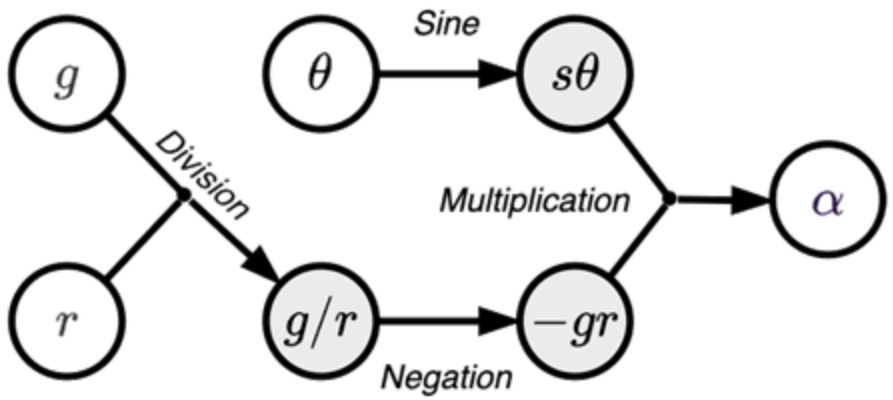




$$\alpha = -\frac{g}{r}\sin\theta$$

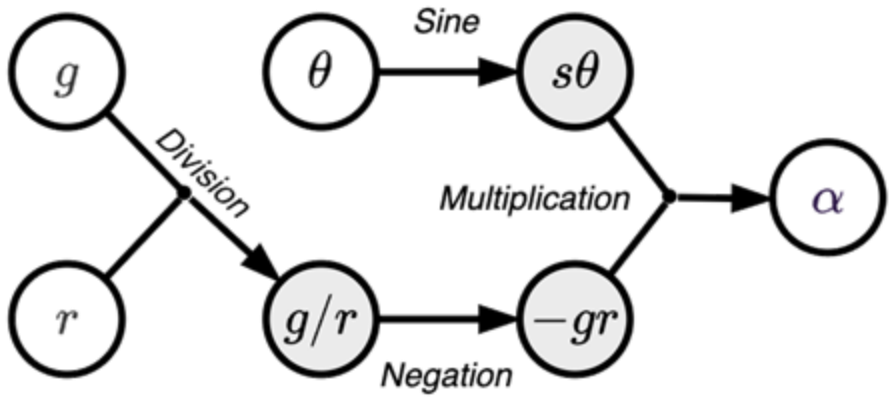
Add Nodes

```
g = Node('gravity', -10)
r = Node('radius', 0.5)
theta = Node('theta')
F = Node('gravitational force')
omega = Node('omega')
alpha = Node('alpha')
time_step = Node('time_step', .03)
time = Node('time', 0)
```



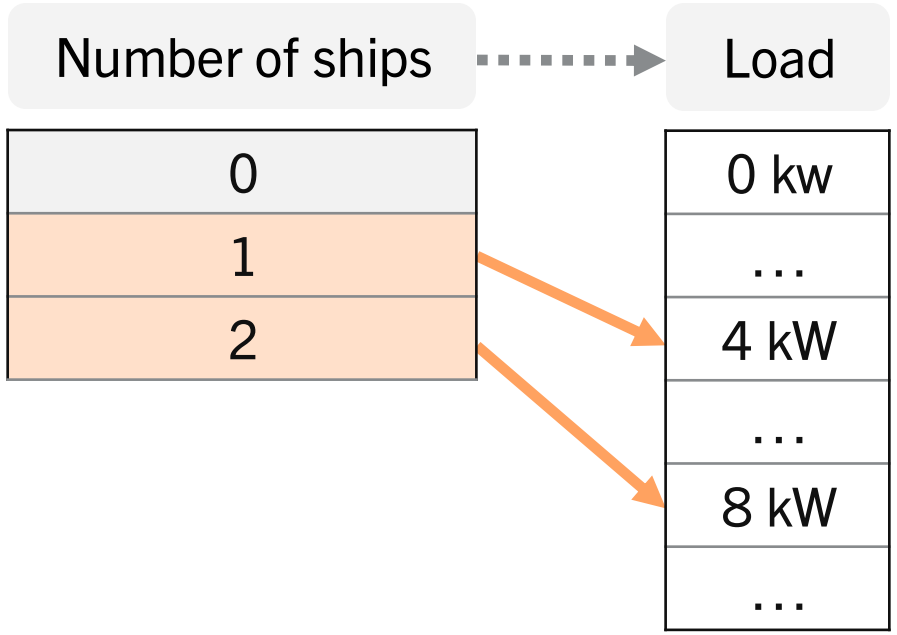
Add Edges

```
hg.add_edge(
   {'s1': g, 's2': r}, target='g/r',
   rel=R.Rdivide,
)
hg.add_edge(
   theta, target=s_theta,
   rel=R.Rsin,
)
hg.add_edge(
   {s_theta, 'g/r'}, target=F,
   rel=R.Rmultiply,
)
hg.add_edge(
   source=F, target=alpha,
   rel=R.Rmean,
   index_offset=1,
)
```



Handling model validity frames

Models with given validity frames can be described by functions that map from a *subset* of a node's values

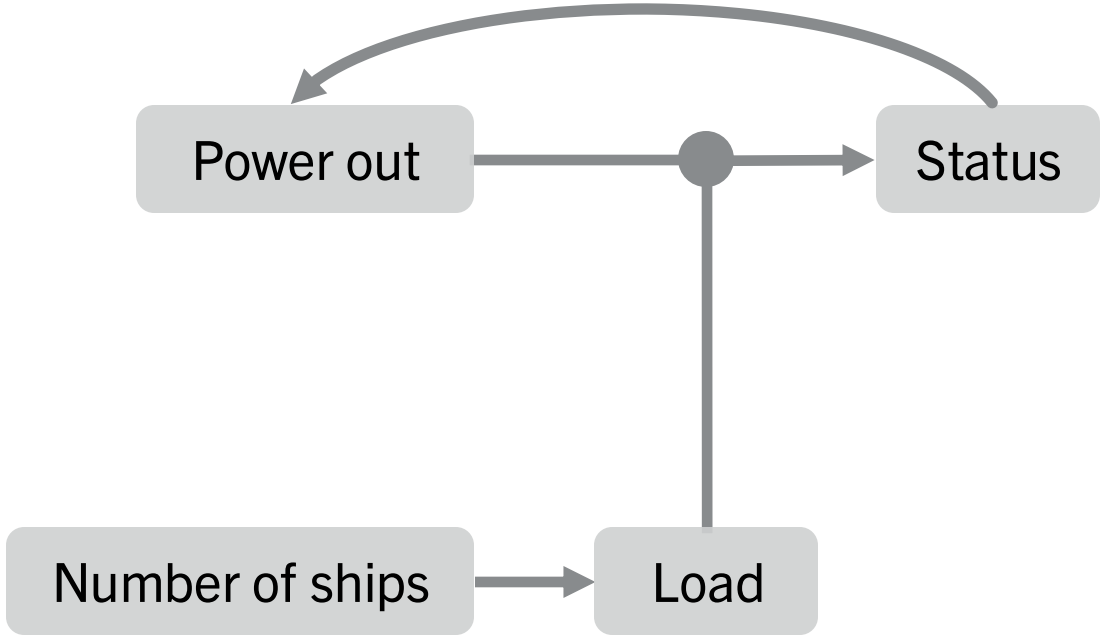


Morris, J., Mocko, G., & Wagner, J. (2025). Unified System Modeling and Simulation via Constraint Hypergraphs. Journal of Computing and Information Science in Engineering, [In Press]

Cycles violate transitivity, indicating conflicting models

Can be dealt with in two ways:

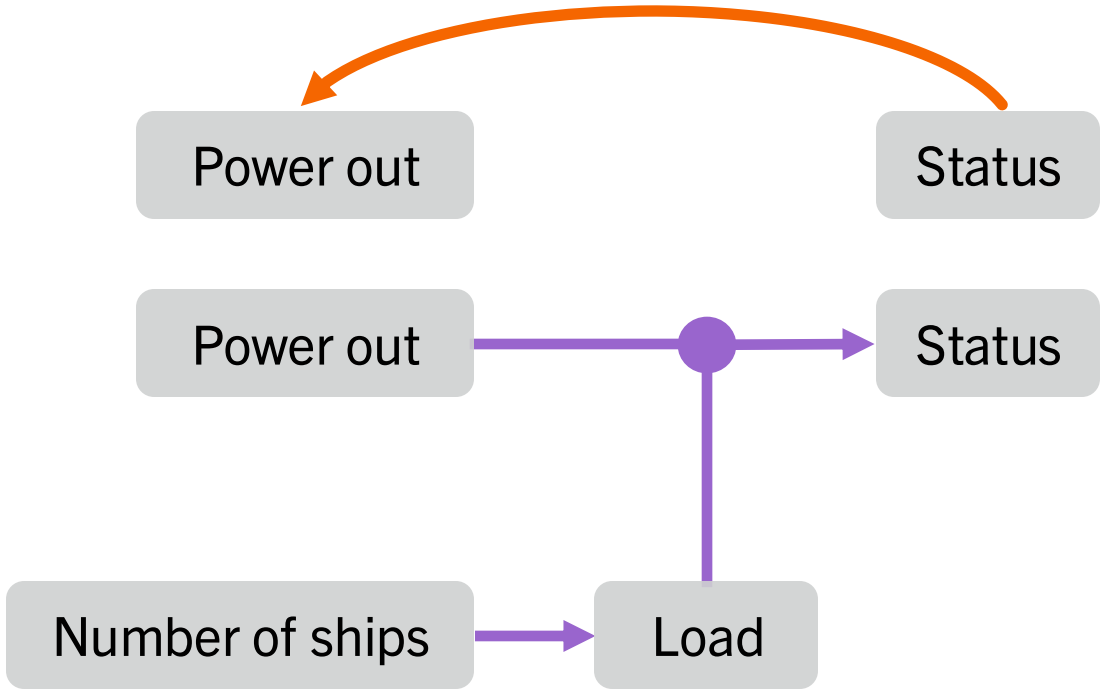
1. Separation
2. ~~Cloning~~ ***Unraveling (Goren-Roig, 2025)***



Morris, J., Mocko, G., & Wagner, J. (2025). Effects of Functional and Declarative Modeling Frameworks on System Simulation. [Manuscript Submitted for Review to the ASME Journal of Dynamic Systems, Measurement and Control].

Separating cycles into distinct graphs

Good for isolating *competing* models

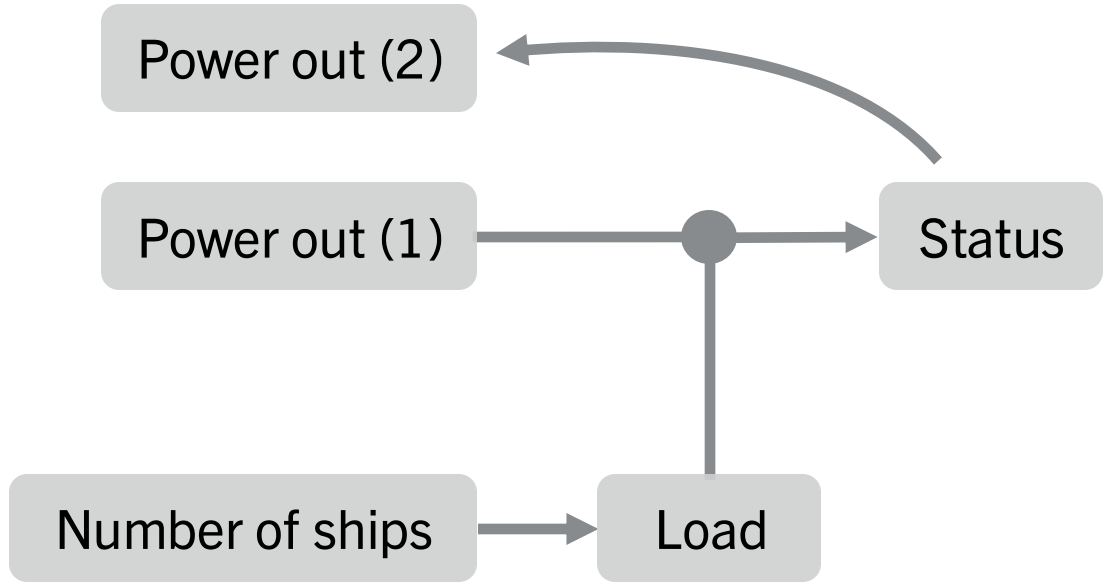


Robinson, M. (2017). Sheaf and Duality Methods for Analyzing Multi-Model Systems. In Recent Applications of Harmonic Analysis to Function Spaces, Differential Equations, and Data Science. Springer International Publishing.

~~Cloning~~ *Unraveling* nodes allows cycles to extend the hypergraph

Good for *pattern* adaption

~~Cloning~~ *Unraveling* occurs while searching for simulation paths and requires an exit edge



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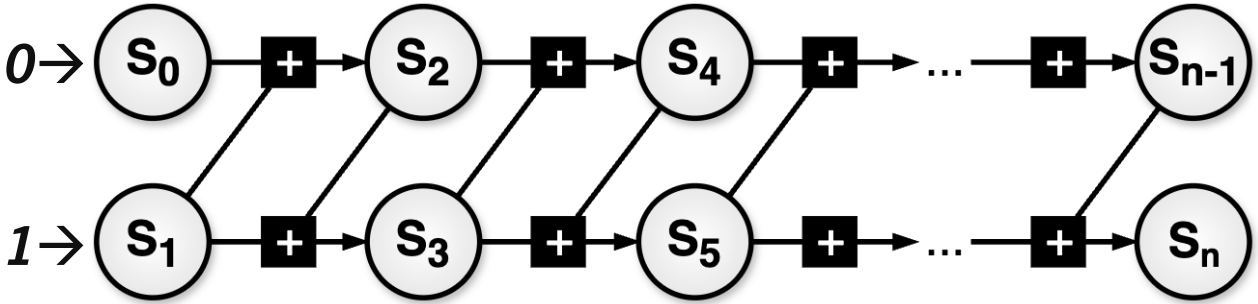
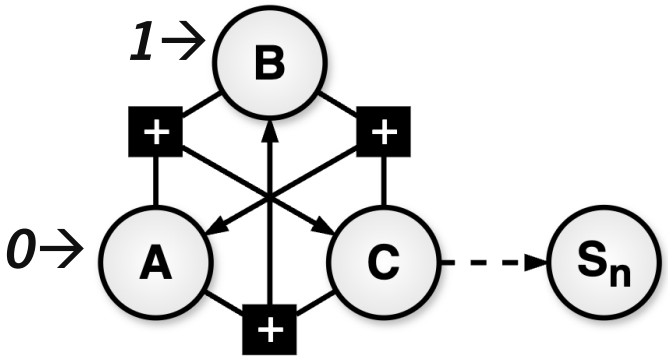
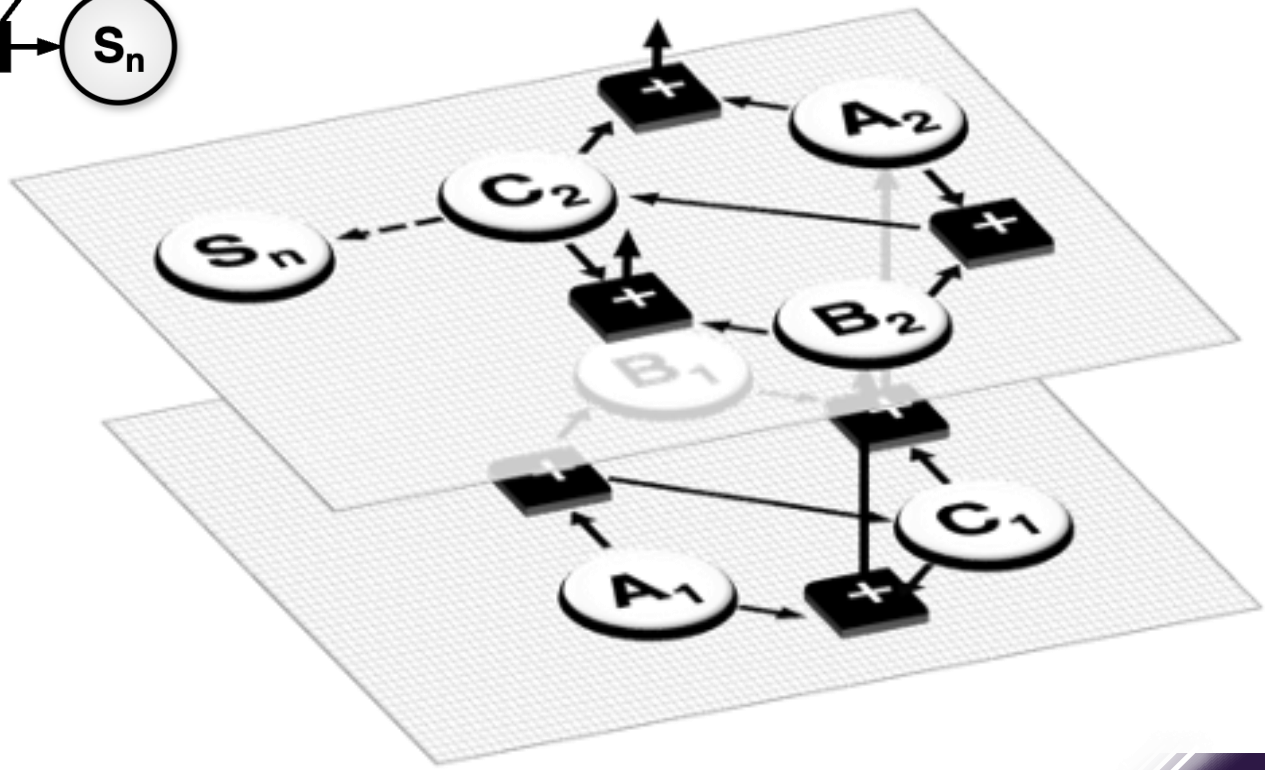
Example computing Fibonacci sequence using cloning

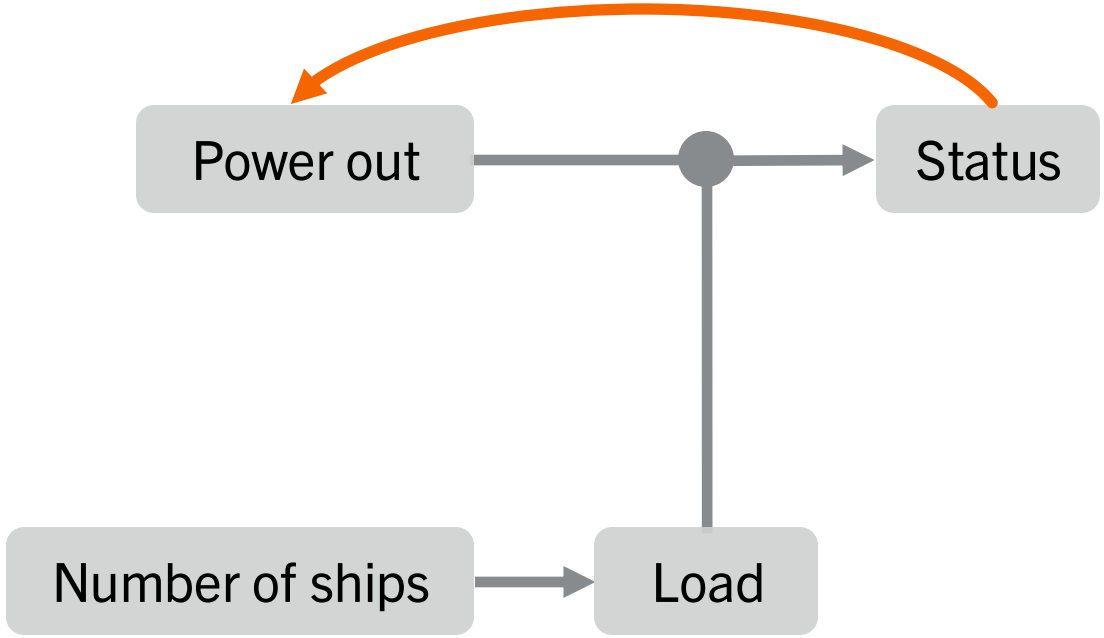






Independent behaviors make the hypergraphs immediately modifiable

Functions can be rewritten, removed, or added without affecting the consistency of the graph *(no side effects)*

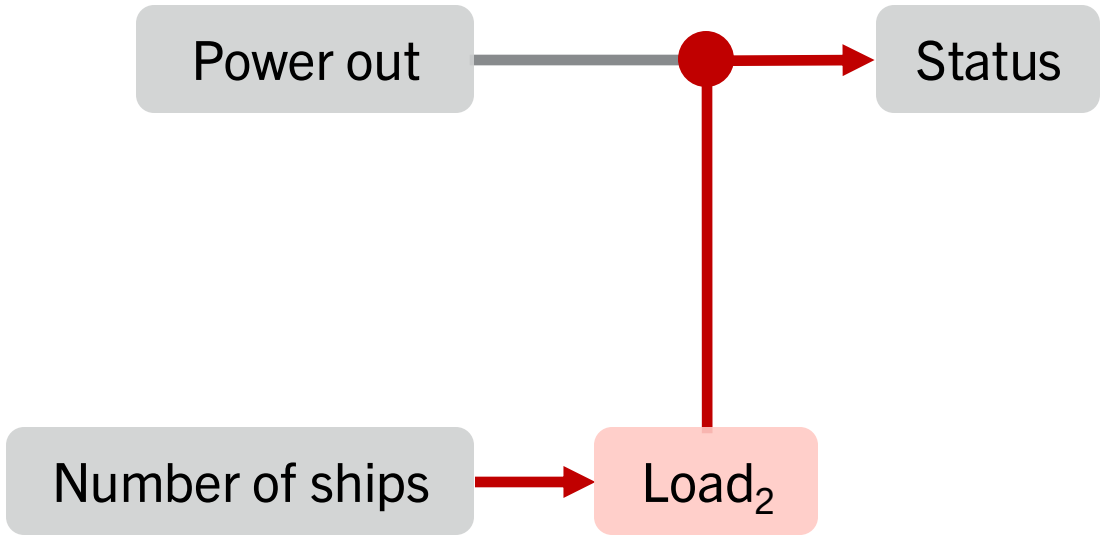


Morris, J., Mocko, G., & Wagner, J. (2025). Effects of Functional and Declarative Modeling Frameworks on System Simulation. [Manuscript Submitted for Review to the ASME Journal of Dynamic Systems, Measurement and Control].

Caveat: scope changes are not guaranteed to be consistent

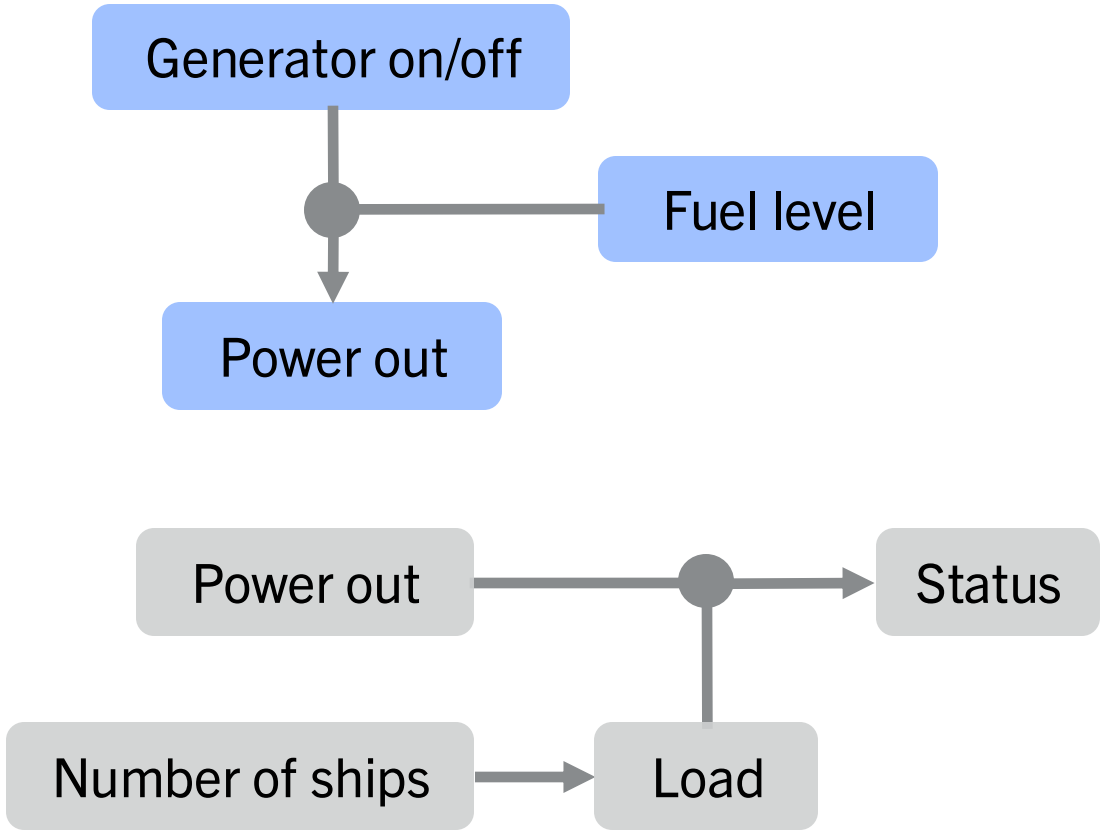
Nodes can be modified only if all connecting edges are also updated to maintain consistency *(only local side effects)*

Behavioral assumptions may be affected



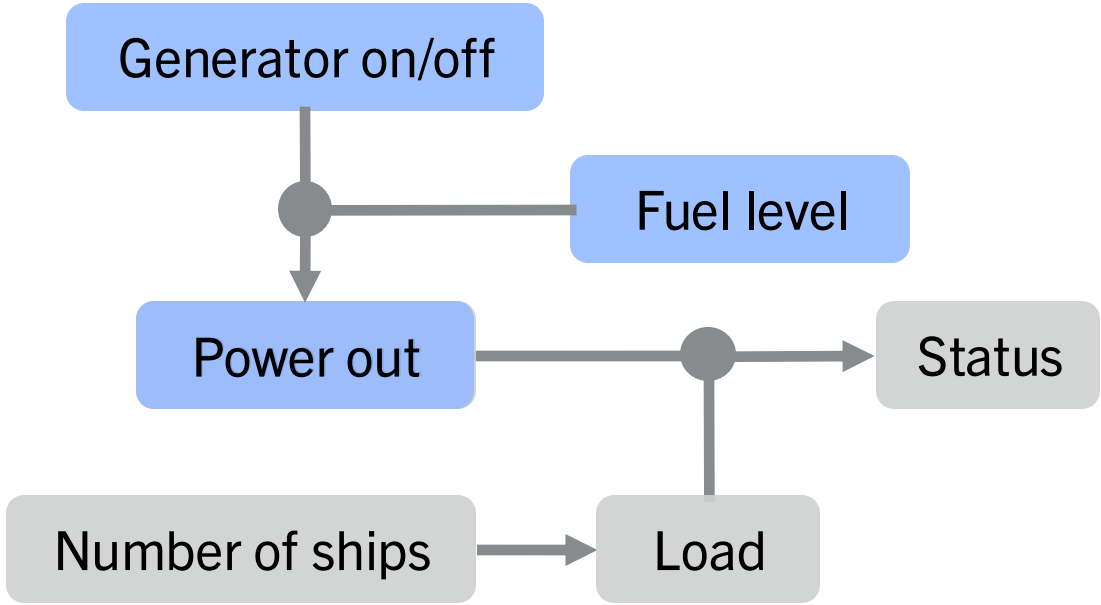
Composable with other graphs along shared nodes (union)

All simulation paths remain consistent *if* new scope does not violate behavioral assumptions



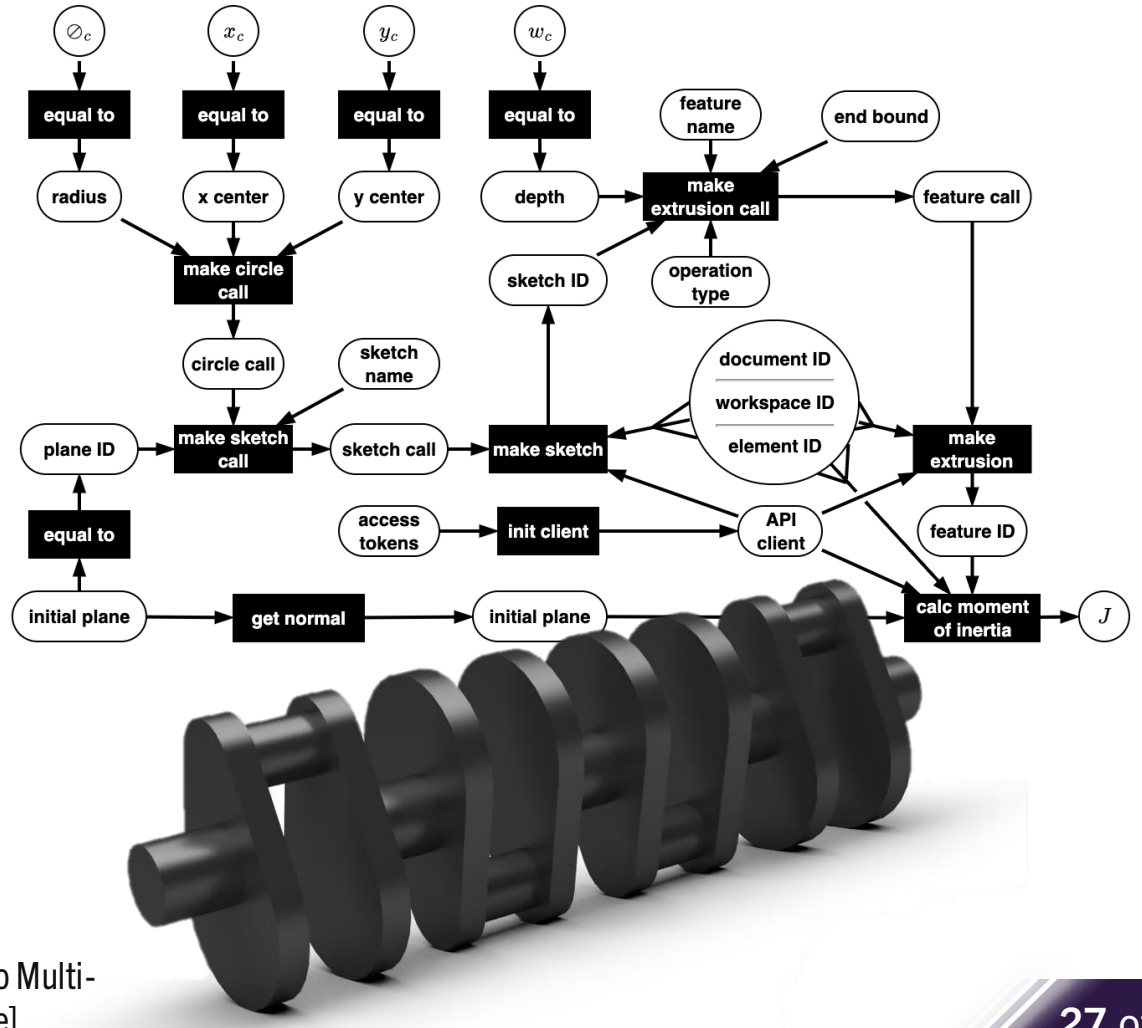
Emergent behavior is provided entirely from composition

All simulation paths remain consistent *if* new scope does not violate behavioral assumptions



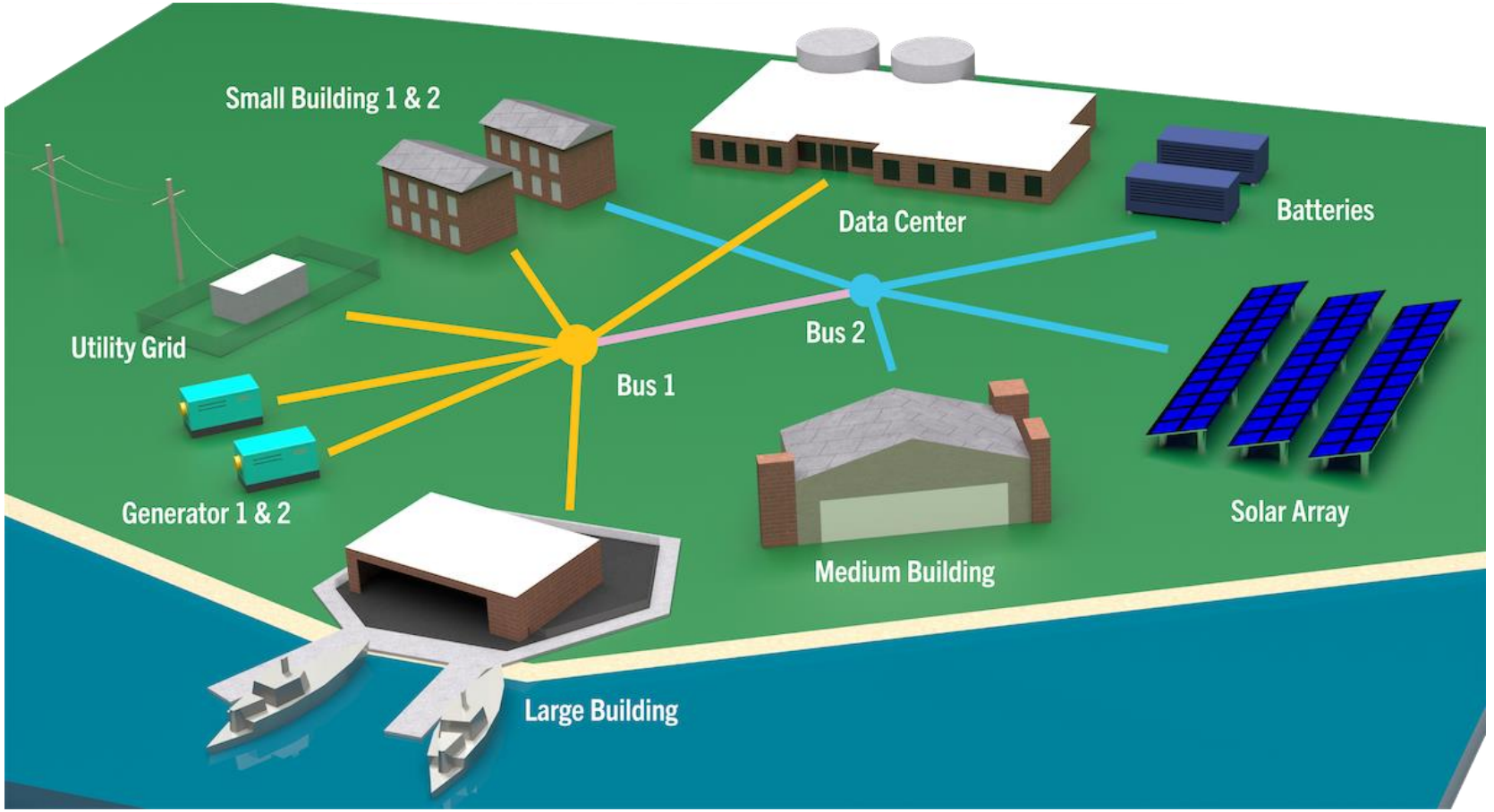
Constraint hypergraphs tie siloed software into a single representation

Software can be integrated by wrapping API calls as edges



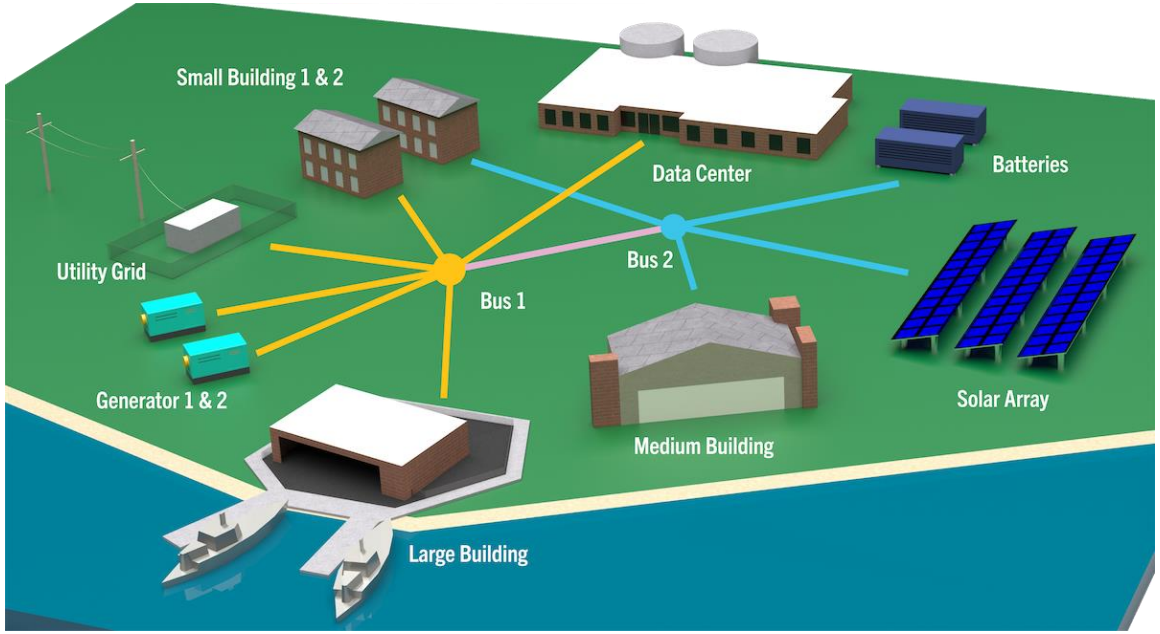
Morris, J., Mocko, G., Wagner, J., & Ramnath, S. (2025). Declarative Integration of CAD Software into Multi-Physics Simulation via Constraint Hypergraphs. [Submitted to the ASME IDETC-CIE 2025 conference]

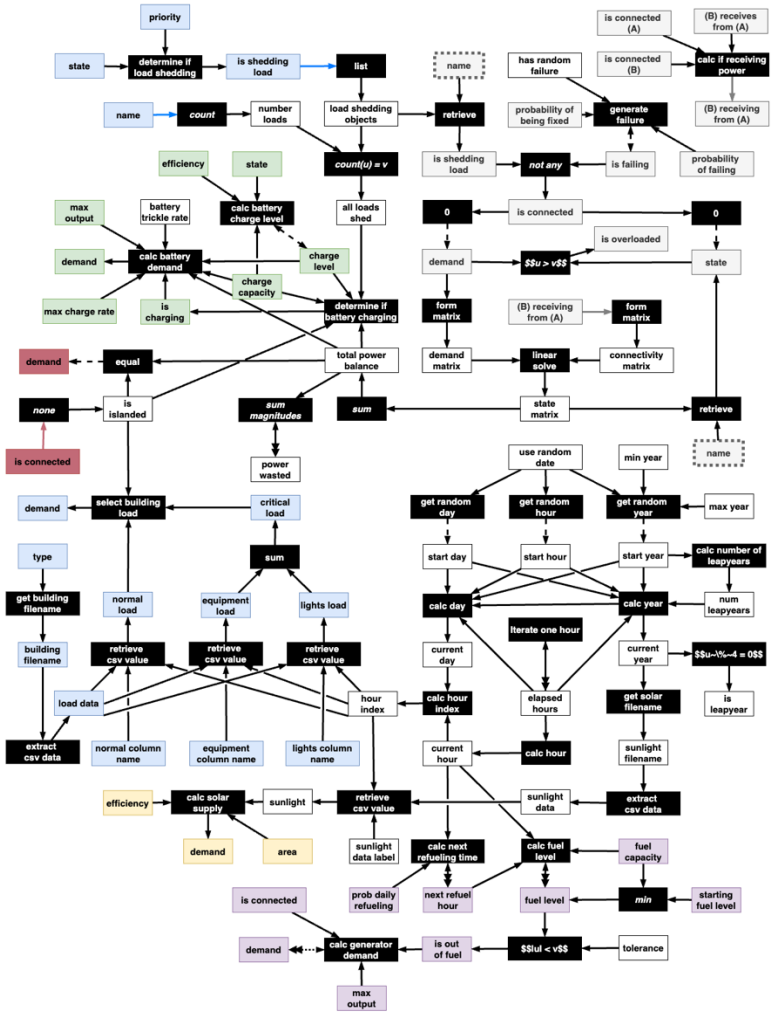
Demo: Integrating more complex systems



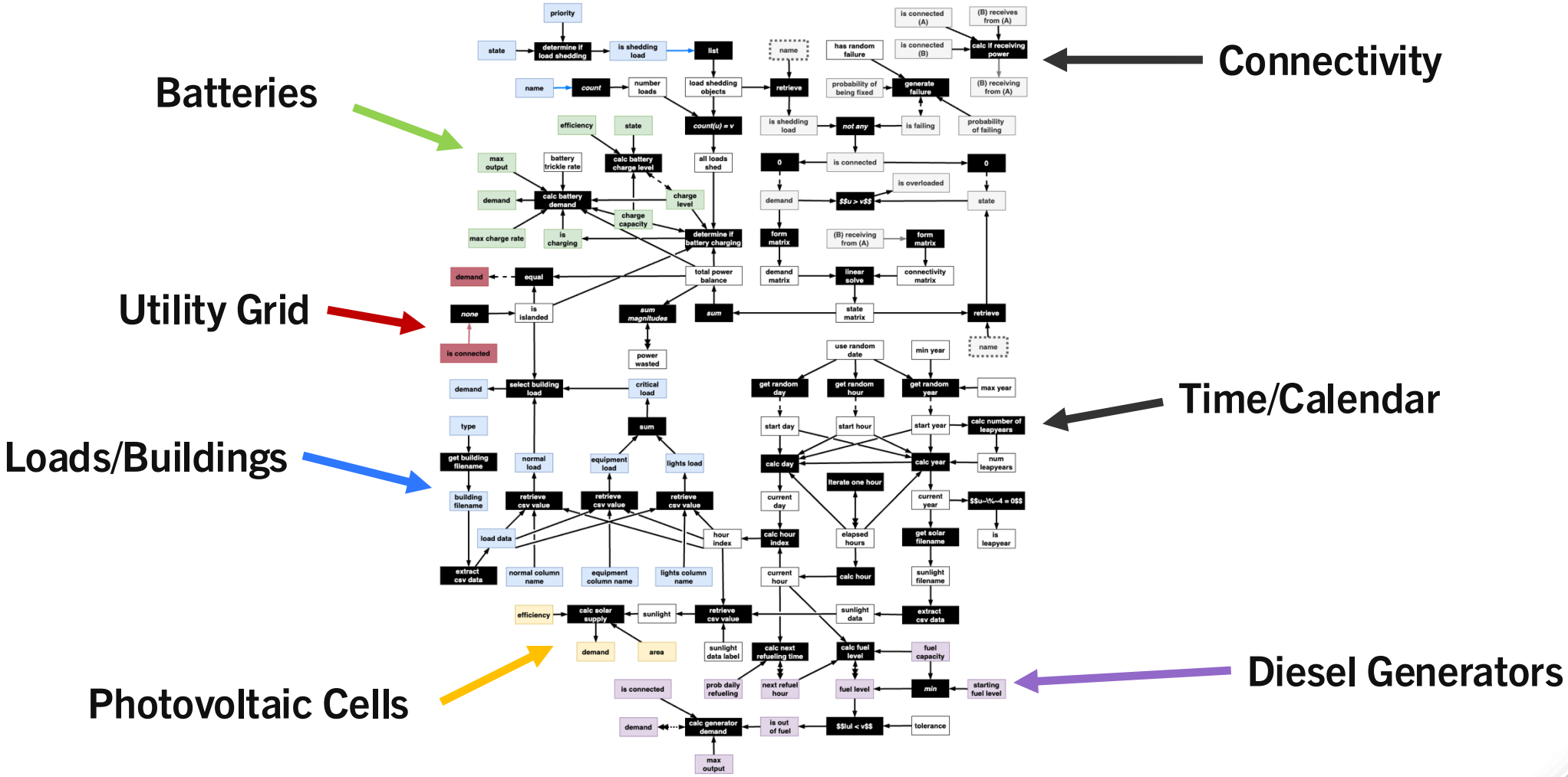
Example available on GitHub by following QR code in corner

Hypergraph contains 594 nodes and 334 edges

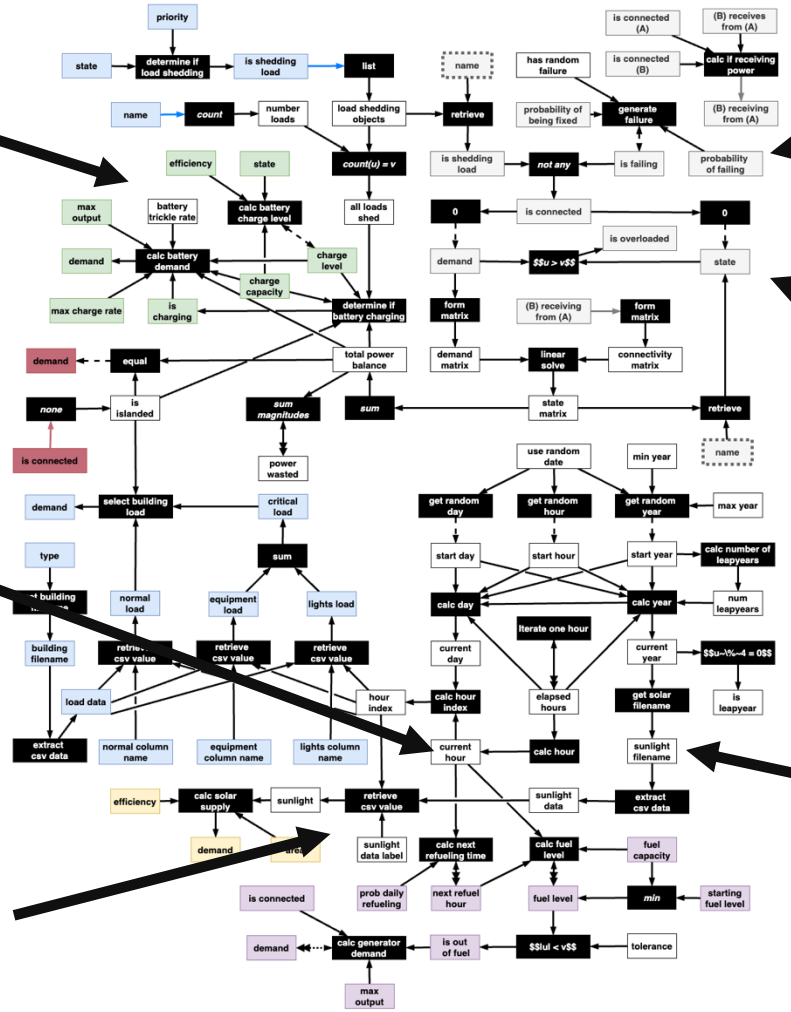




Regions of the hypergraph represent different *things* in the system



Includes all kinds of simulation and modeling



Algebraic rules

Stochastic models

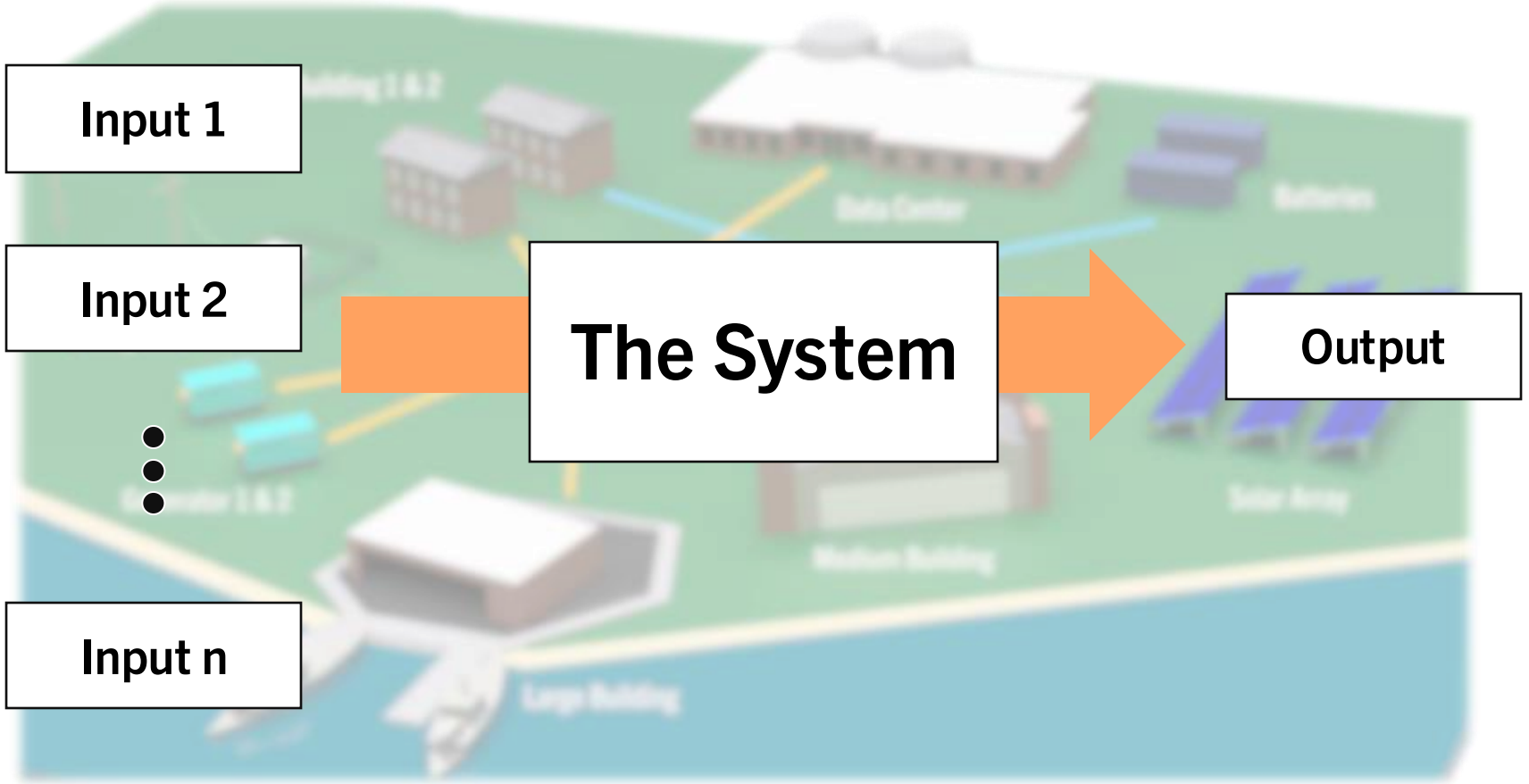
Linear equation solvers

Dynamic simulation

File input/output

Database queries

Hypergraphs of behavioral constraints capture the relationships of things



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Please reach out to jhmrrs@clemson.edu

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